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The Remanufacturing Process of Defense Assets with Stochastic Yield

4 May 2010

Geraldo Ferrer, Associate Professor

Graduate School of Business & Public Policy

Naval Postgraduate School

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Abstract

This study provides the analysis of the material planning strategies for six remanufacturing settings that are common to defense assets and to other technical equipment. These settings are classified according to two dimensions: the supplier's lead time to deliver replacement components (compared to the duration of the process), and the stage in the process that the recovery yield is identified (before disassembly, after disassembly or during component repair). The production plans for the three settings with short supplier's lead time are solved optimally. The long supplier's lead time of the three other settings complicate the analysis, so the paper offers production plans that approximate the optimal policy in these cases.

Keywords: remanufacturing, process yield, material recovery, stochastic optimization, simulation

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About the Author

Geraldo Ferrer is an Associate Professor of Operations Management at the Naval Postgraduate School. Prior to joining NPS, he was in the faculty of the University of North Carolina. His areas of expertise include global operations, supply chain management, sustainable technologies, product stewardship, reverse logistics and remanufacturing. He has also studied inventory problems affecting products made in small batches for frequent deliveries.

He has published on these topics in *European Management Journal, Management Science, Naval Research Logistics, IIE Transactions, Production and Operations Management, European Journal of Operational Research, International Journal of Production Economics, Ecological Economics, Business Horizons, Journal of Operations and Supply Chain Management* and *Resources Conservation and Recycling*. He is a contributor in the *Handbook of Environmentally Conscious Manufacturing* and *Handbook of Industrial Ecology*.

Dr. Ferrer serves as reviewer in many academic journals, the National Science Foundation and the Social Sciences and Humanities Council of Canada. He received a PhD in Technology Management from INSEAD, an MBA from Dartmouth College, a mechanical engineering degree from the Military Institute of Engineering in Rio de Janeiro and a BA in Business Administration from Federal University of Rio de Janeiro. He was founder and director of Superserv Ltd., a company that promoted technology transfer ventures between North American and Brazilian business, introducing innovative technology products for the petroleum industry.

Geraldo Ferrer
Graduate School of Business and Public Policy
Naval Postgraduate School
Monterey, CA 93943-5000
Tel: 831-656-3290
Fax: (831) 656-3407
E-mail: gferrer@nps.edu

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ACQUISITION RESEARCH SPONSORED REPORT SERIES

The Remanufacturing Process of Defense Assets with Stochastic Yield

25 March 2010

Geraldo Ferrer, Associate Professor

Graduate School of Business & Public Policy

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I.   Introduction

Remanufacturing processes have two aspects that stand out and distinguish them from manufacturing: the need for an inventory of used products for recovery (the cores), and the uncertainty of the recovery yield. Core availability is an important constraint that affects the final cost of the remanufacturing operation, with impact on its viability. Below a certain inventory level, the economies of scale may be compromised, and it may be preferable to recycle the used product for its materials. Recovery yield, the second aspect, affects planning because of its influence in the need to purchase replacement parts. Clearly, higher yields lead to more profitable remanufacturing processes. However, the reliability and the time when yield information is available may be as important as the yield level itself, as has been demonstrated in some studies (Debo et al., 2006, Ferrer, 2003, Ferrer and Ketzenberg, 2004).

The remanufacturing process can be analyzed in different ways. Here we focus on the sequence of three operating phases: disassembly, component repair, and reassembly. This study looks at the interaction between the event times marking the beginning of each of these phases, and the moment when the operator observes the recovery yield. Our objective is to determine efficient production plans that reflect the delay in observing yield information in each of six different scenarios of yield information and supplier responsiveness.

There is substantial literature on remanufacturing dealing with tactical, operational, and strategic questions. In many ways, remanufacturing has the same broad goals as manufacturing, such as quality, speed, flexibility, and cost (Ferdows and De Meyer, 1990, Rosenzweig and Roth, 2004). Current manufacturing technologies, practices, and processes can and should be used in support of remanufacturing operations. The transfer of relevant best practices between these different operational settings is an important issue. Throughout our analysis, our focus is on the cost reduction of the remanufacturing operation.

This paper is organized as follows: Section 2 presents a brief review of related articles; Section 3 introduces the different scenarios under study; Section 4 analyzes three scenarios using a responsive new-parts supplier; Section 5 analyzes three other scenarios in which the supplier requires a long lead time, thus leading to purchase decisions prior to acquiring yield information about the parts used in the process; and Section 6 concludes the study with a discussion.

II.   Remanufacturing: A Brief Literature Overview

Remanufacturing closes the materials cycle and provides the basis for product recovery and re-use in supply chains. It focuses on value-added recovery, rather than just materials recovery, that is, recycling. An old estimate indicates that there were more than 73,000 firms engaged in remanufacturing in the US, directly employing over 350,000 people (Giuntini & Gaudette, 2003; Lund, 1983). This number continues to increase with new classes of products that are regularly remanufactured, such as electronic and computer equipment, and new markets that depend on recovered products. Remanufacturing has been described as follows:

> [A]n industrial process in which worn out products are restored to like-new condition. Through a series of industrial processes in a factory environment, a discarded product is completely disassembled. Useable parts are cleaned, refurbished, and put into inventory. Then the new product is reassembled from the old and, where necessary, new parts to produce a unit fully equivalent—and sometimes superior—in performance and expected lifetime to the original new product. (Lund, 1983)

Remanufacturing is therefore different from repair operations, since a product is completely disassembled and all parts are returned to as-new condition before reassembly. It is also different from manufacturing because its inputs are under diverse wear patterns that require greater expertise from the workforce to identify the recovery process required for each component or module extracted from the disassembly operation.

Many authors see remanufacturing as a process of growing importance in the overall product lifecycle. There are several reasons for this, including the cost-effectiveness of remanufacturing in some circumstances, and the product take-back laws that mandate manufacturers to bear the burden of disposal at the end of a product's useful life (Mangun & Thurston, 2002). In short, remanufacturing may make good business sense, with producers recovering a profit from remanufacturing that offsets some of the costs of take-back policies instituted in various communities.

The key point is that in every organization, it is useful to conceptualize remanufacturing as a profit-enhancing or cost-reduction activity.

Often remanufacturing may incorporate component upgrades to add new features to the product or to improve compatibility with newer systems (Ayres, Ferrer & Van Leynseele, 1997). This point is particularly important for the DoD, which is frequently engaged in refreshing its hardware stock with new and improved upgrades. Excellent examples are found in the US Army (i.e., Bradley and Abraham armored-vehicles upgrade programs), the US Marine Corps' Harrier upgrade program, periodic updates of the Navy's aircraft carrier fleet, and numerous examples in the USAF (including the recent B52 and KC135 tanker fleets, which were originally built in the 1950s). Formal models justifying the upgrade decision—including time and extent of repair—are not public; this topic clearly warrants further study in the military context.

In the context of job-shop operations, several studies were originated in the Air force Institute of Technology regarding regular and expedited schedule, inventory buffer, and capacity planning in simulated scenarios based on aircraft maintenance depots (Guide, Kraus & Srivastava, 1997; Guide & Srivastava, 1998; Guide, Srivastava & Jayaraman, 1998; Guide, Srivastava & Kraus, 1997; Guide, Srivastava & Spencer, 1996). These studies generally recommend best approaches to schedule the disassembly-repair-reassembly sequence, considering the uncertainty of the remanufacturing process.

Recently, we have seen a substantial thrust in the literature on closed-loop supply chain that is generally concerned with managing the inventory of used cores to meet the needs of the remanufacturing process, either in quantity, quality or both. Recent examples include Choi, Hwang and Koh (2007); Konstantaras and Papachristos (2007); Teunter, Kaparis and Tang (2008); Visich, Li and Khumawala (2007); and Zikopoulos and Tagaras (2007; 2008). The tutorial by Souza (2008) summarizes some of the key components of these models. In most of the above literature, it is assumed that remanufactured and original products are not

distinguishable. For an extensive review of the reverse-logistics literature, an interested reader may refer to Fleischmann, Bloemhof-Ruwaard, Dekker, Van der Laan, Van Nunen and Van Wassenhove (1997); Guide and Van Wassenhove (2009); and Guide (2000).

This paper expands the literature of production planning in remanufacturing environments by dealing with the value and the availability of yield information at the time when critical decisions are made. (Ferrer, 1996) brought attention to this topic with the development of optimal inventory policies under deterministic demand for a remanufactured product with a single major component in the core. This analysis evolved with the study of four scenarios, representing different times when the yield information is acquired (Ferrer, 2003). Ferrer and Ketzenberg (2004) expanded that study by looking at a product with multiple components of value in the remanufactured core. They use a mathematical program to analyze the model, assuming unlimited supply of cores. Ketzenberg, Souza and Guide (2003) also address the value of yield information in a mixed assembly–disassembly operation with two potential configurations: parallel lines and mixed (shared) production lines. Ketzenberg, van der Laan and Teunter (2006) propose a multi-period model to evaluate the value of full information in a closed-loop model, looking for means to reduce one or more types of production uncertainties: demand, core return or recovery yield. Ketzenberg (2009) evaluates a model of new and remanufactured products that are perfect substitutes and that share limited process capacity to satisfy demand.

This study expands the literature by introducing the following features: (1) it analyzes a product with multiple components of interest, (2) it proposes a multi-period solution, (3) it presents optimal or approximate solution to six scenarios, depending on when yield information is known, (4) it can be implemented for any yield distribution. The model presumes unlimited supply of cores and unrestricted production capacity.

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III.   Potential Scenarios

Krikke, le Blanc and van de Velde (2004) classified product returns into the following four categories: End-of-life Returns, Commercial Returns, End-of-use Returns, and Re-usable Items. The difference between these categories is associated with motivation of the return process, the variation in the product quality and reusability, and the quantity uncertainty:

- End-of-life Returns are associated with the collection of items that have lost their usefulness to the user/owner, and that would otherwise be discarded. Generally, these items have minimal economic value and are recycled for their material. Firms that collect their own products at the end of their useful lives are usually concerned with some environmental motivation, such as actual or potential environmental liabilities.

- End-of-use Returns are generally associated with the end of some contractual obligation between the owner/manufacturer and the customer/user of the product. A typical example is the end-of-lease returns. If the product owner is engaged in several such contracts, then he usually performs a thorough inspection, repair, and possibly upgrade to lease or resell the returned products for the highest price. Occasionally, the owner may choose to disassemble and use the components in the production of remanufactured products. Examples include leased cars and Xerox photocopy machines.

- Commercial Returns are usually associated with retail operations covered by a "satisfaction guarantee" clause. These clauses have existed in some form for almost a century, with the advent of mail-order or catalog sales, and have become the norm in e-commerce. In this situation, the original equipment manufacturer (OEM) has a continuous stream of returns (proportional to original sales) that may be remanufactured, if the design permits.

- Re-usable Item Returns are generally associated with reusable packages, including bottles, pallets, printer cartridges, and other specialty containers of all types and sizes.

The product-return classification can be expanded to include two additional categories: Planned Upgrade and Collect to Recovery.

- Planned Upgrade occurs when a component is recalled (because of an operational or safety defect), or when there is an upgrade that provides benefits that are greater than the cost to implement. It is a common feature when remanufacturing is part of a larger program to extend the life of many assets in a fleet. Actual demand information is known before the disassembly decision, and the fleet operator normally designates the exact pace of implementation by indicating the assets that will be upgraded in each time window, according to process capacity. This description fits the asset management programs in the US Department of Defense.

- Collection for Recovery is associated with failed-parts replacement processes, especially for automobiles. Examples include retreaded tires (Ferrer, 1997) and other automobile components (Ferrer & Whybark, 2001). In some ways, it is similar to End-of-use Return, given that the collected asset has significant remanufacturing potential that can be exercised through a dedicated recovery process. The collection of "not ready-for-issue" components (non-RFI) generated by scheduled and unscheduled maintenance of military assets (aircraft and others) is another example.

This study evaluates the impact of information availability in each stage of the production process, focusing on products that are complex and generally built using a modular design. It assumes that a continuous stream of returned products is available for disassembly, providing some economies of scale to the remanufacturing process. These assumptions are consistent with the product returns described as "End-of-use Returns," "Commercial Returns," "Planned Upgrades," and sometimes "Collection for Recovery" in the classification above. Another assumption in the models is that demand is known before the planning process begins. A random variable—reclaim process yield—defines the remanufacturing scenarios according to the moment when information is realized in the process timeline (please see notation in Table 1). Several cases are possible, which are shown in Table 2 . They depend on when certain decisions have to be made (new-part delivery, disassembly, repair, and reassembly), and when the yield information is finally available to the production planner.

The relationship between new-parts delivery lead time and the finished-product deadline drive the relevance of the new-parts supplier. A responsive

supplier guarantees that the remanufacturer has all parts necessary to complete the assembly kits needed to meet final demand. On the other hand, a supplier with long delivery lead time cannot be used as insurance against low yield rates of the recovery process. Rather, orders placed with such supplier can only be used to hedge against the yield variability of some items that make up the assembly kit, but they cannot guarantee against stockouts. This creates two variations for each of the three columns, for a total of six scenarios in this study. The row numbers and column letters in Table 2 identify the scenarios.

In all scenarios, these simplifying assumptions frame the models (although assumptions 3-6 are easy to relax without major changes):

- 1.   Components have two sources: from a perfectly reliable supplier of new parts or components and from the recovery process, which employs used assets as the main source of reparable components. The collection process may be insufficient to generate all the reparable parts needed to meet demand, so the production kit is complemented with new parts.

- 2.   Setup costs are negligible. This is a reasonable assumption if the remanufacturing process operates continually using dedicated resources.

- 3.   There are no capacity constraints in the part-recovery processes. The capacity analysis would change the focus of the model away from supplier responsiveness and timing of yield information, our main concerns.

- 4.   There is an unlimited supply of used products to remanufacture. Consequently, the disassembly lot size is not constrained.

- 5.   Disposal costs are not considered. Generally, the cost of disposal makes repairing even more attractive for all parts.

- 6.   The product has a limited number of components or modules that are relevant for the remanufacturing process, and only one unit of each is need to build the product. The cost of other items in the product is immaterial, and does not affect the remanufacturing decision.

In Section 4, we analyze the scenario variations in which the manager can wait until recovery yield information is available before ordering new parts to replace

damaged ones. In Section 5, we look at the cases which the supplier's lead time is long, causing the planner to order new parts before yield information is known. Section 6 concludes with a discussion and suggestions for future research.

Table 1.   Notation

Objective Function		Cost parameters	
$C(N, X, Q)$	remanufacturing cost per period;	k	disassembled cost per machine;
Decision Variables		s	shortage cost per machine;
N_t	used machines disassembly lot size;	g_i	holding cost per repaired or new part per period;
X_t, x_{it}	new-parts procurement; X_t is the matrix of all x_{it};	h_i	holding cost per reparable part per period;
Q_t, q_{it}	used parts lot size; Q_t is the matrix of all q_{it};	r_i	repair cost per part;
v_{it}	disassembly lot size to meet the demand for item i;	p_i	new-part procurement price;
State Variables		**Other**	
l_t	inventory of cores at the end of the period;	z	number of distinct parts in finished product;
Ω_t, ω_{it}	inventory of new parts at the end of period t; Ω_t is the matrix of all ω_{it};	$\lceil \upsilon \rceil$	$\min\{a \in \text{Integers}: a \geq \upsilon\}$
M_t, m_{it}	inventory of reparable parts at the end of period t; M_t is the matrix of all m_{it};	$\lfloor \upsilon \rfloor$	$\max\{a \in \text{Integers}: a \leq \upsilon\}$
Θ_t, θ_{it}	finished product shortage at the end of period t; Θ_t is the matrix of all θ_{it};	υ^+	$\max\{\upsilon, 0\}$
Event Times		**Random Variable**	
t_X	decision time of parts purchase quantity;	Y_t, y_{it}	reclaim process yield; Y is the matrix of all y_i;
t_N	decision time of disassembly quantity;		
t_Q	decision time of repair quantity;		
t_Y	time when yield information is realized;		

Table 2.   Remanufacturing Scenarios

	A. early yield information	B. yield information upon disassembly	C. yield information upon repair
1. responsive supplier	$t_Y \leq t_N < t_Q$ $t_Y \leq t_X$	$t_N < t_Y \leq \text{And}\{t_Q, t_X\}$	$t_N < t_Q < t_Y \leq t_X$
2. long delivery supplier	$t_X < t_Y \leq t_N < t_Q$	$\text{And}\{t_X, t_N\} < t_Y \leq t_Q$	$t_N < t_Q < t_Y$ $t_X < t_Y$

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IV.   Analysis: Responsive New-parts Supplier

Since the yield is known before the planner makes disassembly decisions, it is possible to avoid component shortage altogether. Therefore, in this scenario, the cost function includes just disassembly, purchase, repair, and holding costs (there are no shortage costs).

$$C(N_t) = kN_t + \sum_i \left[p_i x_{it} + r_i q_{it} + g_i \omega_{it} + h_i m_{it} \right] \tag{1.}$$

Early Yield Information

If the manager acquires the yield information before core disassembly starts, then we have one of the scenarios in column A of Table 2. In addition to the cases in the Scheduled Upgrade category (see Section III), it is possible to know yield before actual disassembly and inspection of parts by using efficient lifecycle tracking systems.

The assets that are taken to Scheduled Upgrades are mostly operational. They are brought to the remanufacturing depot where outdated components are replaced (yield = 0%) and other components are repaired with nearly 100% yield. If the organization uses an efficient lifecycle tracking system, such as item-unique identification (IUID), or if item maintenance is properly recorded, the history of each asset can help in determining in advance which parts need replacement or not, effectively revealing the yield. (For an analysis on the challenges related to lifecycle tracking of assets, see (Apte and Ferrer, 2010)). Since the component recovery yield and the demand information are known before production planning is started, the analysis is deterministic, and relatively trivial. The sequence of events is the following:

1.   Learn yield (y_i) of each part for the disassembly kit.

2.   Disassemble N used machines, discarding the components that will not be repaired and reused.

3. Place purchase order for x_i parts towards obtaining D kits for the reassembly process.

4. Repair q_i of each reparable part i.

5. Receive x_i parts ordered from new-parts supplier.

6. Deliver exactly D kits of each part to the reassembly line.

It is usually true that holding cost for repaired items is higher than for reparable items. Therefore, there is no need to hold excess inventory of new or repaired items at the end of the period. Consequently, the stock of repaired parts at the end of the period is just

$$\omega_{it} = \left(\omega_{i,t-1} - D\right)^+$$

Figure 1 shows the decision process, with emphasis on the critical yield of each part: the lowest recovery rate necessary to meet the demand for the respective part in the period.

When the planner schedules the parts-repair process, the recovery yield is already known, so it is possible to schedule the precise number of cores needed to build the right number of reassembly kits. Likewise, it is possible to order the exact number of new parts to meet demand. The optimal number of items repaired and the number of items purchased are functions of the number of cores disassembled.

$$x_{it} = \left(D - \omega_{i,t-1} - y_{it}N_t - m_{i,t-1}\right)^+ \tag{2.}$$

$$q_{it} = \min\left\{y_{it}N_t + m_{i,t-1}, D - \omega_{i,t-1}\right\} \tag{3.}$$

```
                                                    ┌─────────────────────────────────────┐
                                                    │  Do not buy or repair any part type i.│
        ◇───────────────────────────◇              │  Supply demand for part i from inventory.│
       Is there sufficient inventory of type i      │  Reparable parts inventory of type i increases│
        repaired parts to meet demand?    ── yes ▶ │  according to the number of cores disassembled.│
        ◇───────────────────────────◇              └─────────────────────────────────────┘
                      │
                      no
                      ▼
        ┌───────────────────────────────────┐
        │  Final repaired parts inventory of type i is zero.│
        └───────────────────────────────────┘
                      │
                      ▼
        ◇───────────────────────────◇
       Is there sufficient inventory of type i          ┌─────────────────────────────────────┐
        reparable parts to meet demand?    ── yes ──▶  │  Repair type i parts from inventory as needed to│
        ◇───────────────────────────◇                 │  meet demand.│
                      │                                │  Do not buy new type i parts.│
                      no                               │  Reparable parts inventory of type i changes│
                      ▼                                │  according to the number of cores disassembled│
        ◇───────────────────────────◇    ── yes ──▶  │  and number of parts repaired.│
       Is actual yield for part i greater than         └─────────────────────────────────────┘
        the critical yield?
        ◇───────────────────────────◇
                      │
                      no
                      ▼
        ┌───────────────────────────────────┐
        │  Repair all type i parts obtained from disassembly +│
        │  reparable parts in stock.│
        │  Buy additional new parts type i to meet demand.│
        │  Final repairable parts inventory or type i is zero.│
        └───────────────────────────────────┘
```

Figure 1.   **Rep air and Purchase Quantity Policies in Scenarios A-1 and B-1**

The residual inventory of reparable parts—items obtained from disassembly but that are not needed to meet demand in this period—is the main source of holding cost. When there is excess inventory of reparable items, there is no need to buy more of that item to complement the reassembly kit:

$$m_{it} = \left(y_{it} N_t + m_{i,t-1} - \left(D - \omega_{i,t-1} \right) \right)^+ \tag{4.}$$

The cost function can be easily minimized by finding the minimum number of cores necessary to disassemble in order to fulfill the demand for each part i entirely from repairs:

$$v_{it} = \left\lceil \frac{D - \omega_{i,t-1} - m_{i,t-1}}{y_{it}} \right\rceil$$

It is not necessary to consider any part for which $v_{it} \leq 0$, because these items already have sufficient inventory of repaired or reparable units to meet demand. To find the optimal policy, it requires to rename and rank the remaining parts such that

$$v_{it} < v_{jt} \Rightarrow 1 \leq i < j \leq z$$
.

Let $0 \leq N_t \leq v_{1,t}$. Consequently,

$$C\left(N_t | X_t, Q_t\right) = kN_t + \sum_i \left[p_i \left(D - \omega_{i,t-1} - y_{it}N_t - m_{i,t-1} \right) + r_i \left(y_{it}N_t + m_{i,t-1} \right) \right]$$

$$C\left(N_t | X_t, Q_t\right) = N_t \left(k - \sum_i y_{it} \left(p_i - r_i \right) \right) + \sum_i \left[p_i \left(D - \omega_{i,t-1} \right) - \left(p_i - r_i \right) m_{i,t-1} \right] \tag{5.}$$

Considering that the second term is constant in N_t, we have

$$\left. \frac{\partial C\left(N_t\right)}{\partial N_t} \right|_{N_t \leq v_{1,t}} = k - \sum_i y_{it} \left(p_i - r_i \right) \tag{6.}$$

If the first derivative given by equation (6) is positive, then no core should be disassembled. Moreover,

$$x_{it}{}^* = D - \omega_{i,t-1} - m_{i,t-1} \tag{7.}$$

$$q_{it}{}^* = m_{i,t-1} \tag{8.}$$

and the residual inventory of reparable parts is just $m_{it} = \left(m_{i,t-1} - \left(D - \omega_{i,t-1}\right)\right)^+$. If the first derivative equals zero, then the decision-maker is indifferent between disassembling any quantity between 0 and $v_{1,t}$. If the first derivative is negative, then the cost function reduces with larger disassembly lots, and $N_t^* \geq v_{1,t}$. Now, let $v_{j,t} \leq N_t \leq v_{j+1,t}$ and $1 \leq j$. Then,

$$C\left(N_t \middle| X_t, Q_t\right) = kN_t + \sum_{i \leq j}\left[r_i\left(D - \omega_{i,t-1}\right) + h_i\left(y_{it}N_t + m_{i,t-1} - D + \omega_{i,t-1}\right)\right]$$
$$+ \sum_{i > j}\left[p_i\left(D - \omega_{i,t-1} - y_{it}N_t - m_{i,t-1}\right) + r_i\left(y_{it}N_t + m_{i,t-1}\right)\right]$$

$$C\left(N_t \middle| X_t, Q_t\right) = N_t\left[k + \sum_{i \leq j}y_{it}h_i - \sum_{i > j}y_{it}\left(p_i - r_i\right)\right]$$
$$+ \sum_{i \leq j}\left[\left(r_i - h_i\right)\left(D - \omega_{i,t-1}\right) + h_i m_{i,t-1}\right] + \sum_{i > j}\left[p_i\left(D - \omega_{i,t-1}\right) - \left(p_i - r_i\right)m_{i,t-1}\right] \tag{9.}$$

There is no holding cost for repaired or new items because there is no excess inventory of these items, as shown in Figure 1. However, there are $y_{it}\left(N_t - v_{it}\right)$ reparable items of type $i \leq j$ left at the end of the period. Since only the first term in the cost function depends on N_t, then

$$\left.\frac{\partial C\left(N_t\right)}{\partial N_t}\right|_{v_{j,t} \leq N_t \leq v_{j+1,t}} = k + \sum_{i \leq j}y_{it}h_i - \sum_{i > j}y_{it}\left(p_i - r_i\right) \tag{10.}$$

Let $j^* = \max\left\{j : k + \sum_{i \leq j}y_{it}h_i \leq \sum_{i > j}y_{it}\left(p_i - r_i\right)\right\}$. Hence, $\left.\frac{\partial C\left(N_t\right)}{\partial N_t}\right|_{v_{j^*,t}} \leq 0$ and

$\left.\frac{\partial C\left(N_t\right)}{\partial N_t}\right|_{v_{j^*+1,t}} > 0$. Since $C(N_t)$ is convex, $N_t^* = v_{j^*,t}$, it minimizes the cost function.

Clearly, $\forall i < j^*$, $m_{it} > 0$. Moreover,

$$q_{it}^* = \begin{cases} D - \omega_{i,t-1} & i \leq j^* \\ y_{it}v_{j^*,t} + m_{i,t-1} & i > j^* \end{cases} \tag{11.}$$

$$x_{it}^* = \begin{cases} 0 & i \leq j^* \\ D - \omega_{i,t-1} - y_{it}v_{j^*,t} - m_{i,t-1} & i > j^* \end{cases} \qquad (12.)$$

Applying equations (11) and (12) to (4), we have the residual inventory of reparable parts:

$$m_{it} = \begin{cases} y_{it}v_{j^*,t} + m_{i,t-1} - \left(D - \omega_{i,t-1}\right) & i \leq j^* \\ 0 & i > j^* \end{cases}$$

Yield Information upon Disassembly

The most common situation in a remanufacturing process is probably the first scenario in column B of Table 2. A simple test of the components released during disassembly—either a visual inspection or with the aid of basic diagnostic tools—allows for precise determination of whether the components are suitable for recovery. This is the sequence of events:

Disassemble N machines.

Learn yield y_i of each part during disassembly.

Discard unrecoverable parts, saving $y_i N$ of each part i.

Place purchase order for x_i parts towards obtaining D kits for the reassembly process.

Repair q_i of each reparable part i.

Receive x_i parts ordered from new-parts supplier.

Deliver exactly D kits of each part to the reassembly line.

When the planner schedules parts to recover, yield information is already known. He may order new parts to complement the reassembly kit at this moment, especially those with low yield. Since recovered parts are less expensive than purchased parts, only parts needed immediately are ordered. The optimal number of parts repaired and the number of parts purchased are functions of the number of cores disassembled, given by equations (2) and (3) in the previous section. The residual inventory of reparable parts is also the same as in equation (4). The same

Figure 1 shows the decision process. This time, however, disassembly lot size is decided before the yield information is known. Let $F_i(y)$ be the probability that the yield of part i is less than y, and $f_i(y)$ is its density function. Hence, applying equations (2)-(4) into the cost function, after some manipulation, we have:

$$C(N_t|X_t, Q_t) = kN_t + \sum_i \int_0^{\frac{D-\omega_{i,t4}-m_{i,t4}}{N_t}} \left((r_i - p_i)(yN_t + m_{i,t-1}) + p_i(D - \omega_{i,t-1}) \right) dF_i(y)$$
$$+ \sum_i \int_{\frac{D-\omega_{i,t4}-m_{i,t4}}{N_t}}^1 \left((r_i - h_i)(D - \omega_{i,t-1}) + h_i(yN_t + m_{i,t-1}) \right) dF_i(y)$$

(13.)

The cost function is clearly convex. The first sum of integrals is a consequence of new-parts purchase, and the second sum of integrals is a consequence of the excess inventory of high-yield reparable parts. The limit of integration is the critical yield. Using Leibnitz's differentiation rule, we obtain the gradient of the cost function with respect to the disassembly lot size.

$$\frac{\partial C(N_t)}{\partial N_t} = k + \sum_i \left[\int_{\frac{D-\omega_{i,t4}-m_{i,t4}}{N_t}}^1 yh_i dF_i(y) - \int_0^{\frac{D-\omega_{i,t4}-m_{i,t4}}{N_t}} y(p_i - r_i) dF_i(y) \right]$$

(14.)

The expression above shares a similarity with the gradient of the cost expressions in the previous section, equations (6) and (10). Cost is minimized for

$$N_t^* = \max\left\{ N_t \in \; Integers : \frac{\partial C(N_t)}{\partial N_t} \leq 0 \right\}, \quad x_{it}^* = \left(D - \omega_{i,t-1} - y_{it}N_t - m_{i,t-1} \right)^+ \text{ and}$$

$q_{i,t}^* = D - x_{it}^* - \omega_{i,t-1}$. Residual inventories of repaired and reparable parts are

$\omega_{it} = \left(\omega_{i,t-1} - D \right)^+$ and $m_{it} = \left(y_{it}N_t^* + m_{i,t-1} - (D - \omega_{i,t-1}) \right)^+$, respectively.

Yield Information upon Component Repair

Some components, both electric and mechanical, may require expensive testing procedures in order to evaluate their suitability for remanufacturing. The cost of these tests may be impractical, so it may be more reasonable to attempt a joint

test-and-repair process that enables the planner to acquire the yield information about each part. The first scenario in column C of Table 2 shows the decision-time relationships. This is the sequence of events:

Disassemble N machines.

Repair and test each part obtained from the disassembly process.

Learn yield of each part during repair.

Discard unrecoverable parts, saving $y_i N$ of each part i.

Place purchase order for x_i parts towards obtaining D kits for the reassembly process.

Receive x_i parts ordered from new-parts supplier.

Deliver exactly D kits of each part to the reassembly line.

When the planner orders new parts, yield information is already known, so they can be ordered in exact quantities to complement the reassembly kit. As a result, shortages are completely avoided in this scenario, as well. However, contrary to the last two cases, all parts are repaired upon disassembly, leading to higher holding costs. If the stock of a particular item exceeds the demand for that part, then none of it is repaired, and the parts retrieved in disassembly join the stock of reparable parts. Otherwise, it is more practical to repair all parts retrieved during disassembly, in addition to any reparable part in stock, exhausting the inventory of reparable parts in that period. That is:

$$\begin{cases} q_{it}^* = N_t + m_{i,t-1} & m_{it} = 0 & \text{if } D < \omega_{i,t-1} \\ q_{it}^* = 0 & m_{it} = m_{i,t-1} + N_t & \text{if } D \geq \omega_{i,t-1} \end{cases}$$

The number of cores to disassemble depends only on the parts that have insufficient stock of repaired parts to meet the demand in the current period (at that point, the disassembly costs from previous periods are sunk). To find the optimal policy, the analysis needs to consider just the part types that cannot meet demand from inventory in the current cycle, as shown in Figure 2. Those are the parts satisfying $D - \omega_{i,t-1} > 0$. Since repaired parts are less expensive to acquire than

purchased parts, the optimal number of items purchased is a function of the number of cores disassembled and of the recovery yield:

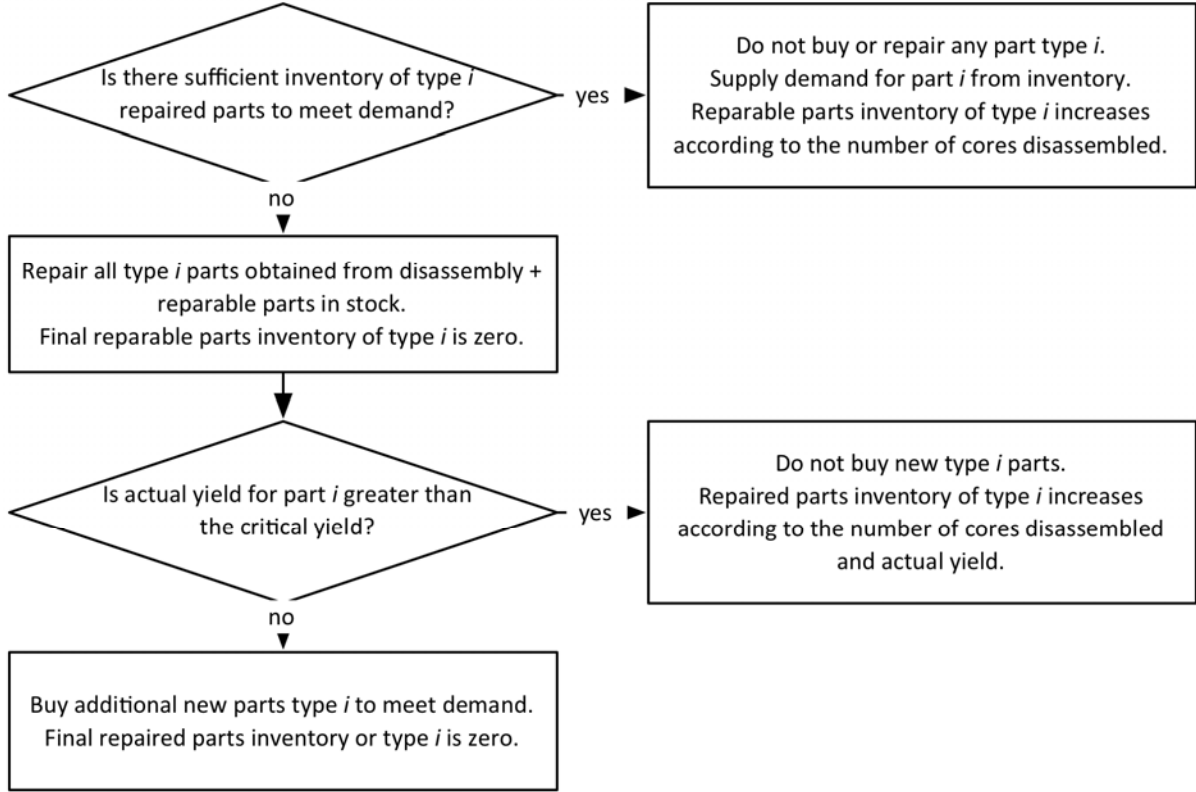$$x_{it}^* = \left(D - \omega_{i,t-1} - y_{it}\left(N_t + m_{i,t-1}\right)\right)^+$$

(15.)

Consequently, the residual inventory of repaired parts (ready-for-issue) is:

$$\omega_{it} = \left(\omega_{i,t-1} + y_{it}\left(N_t + m_{i,t-1}\right) - D\right)^+$$

(16.)

Using equations (15)-(16), after some manipulation, the cost function becomes

$$C\left(N_t|X_t, Q_t\right) = kN_t + \sum_i \left[\int_0^{\frac{D-\omega_{i,t4}}{N_t+m_{i,t4}}} p_i\left(D - \omega_{i,t-1} - y\left(N_t + m_{i,t-1}\right)\right) dF_i(y) \right]$$

$$+ \sum_i \int_{\frac{D-\omega_{i,t4}}{N_t+m_{i,t4}}}^1 g_i\left(y\left(N_t + m_{i,t-1}\right) + \omega_{i,t-1} - D\right) dF_i(y)$$

(17.)

Figure 2. Repair and Purchase Policies in Scenario C-1

Equation (17) has the same structure as the cost function in equation (13). Notice the different critical yield in the limits of integration. Using Leibnitz's rule for differentiation, we obtain the gradient of the cost function with respect to the disassembly lot size:

$$\frac{\partial C(N_t)}{\partial N_t} = k + \sum_i \left[\int_{\frac{D-\omega_{i,t4}}{N_t + m_{i,t4}}}^{1} yg_i dF_i(y) - \int_{0}^{\frac{D-\omega_{i,t4}}{N_t + m_{i,t4}}} yp_i dF_i(y) \right] \qquad (18.)$$

The expression above is similar to the gradient of the cost expression in the previous sections. Cost is minimized for $N_t^* = \max\left\{ N_t \in \; Integers : \frac{\partial C(N_t)}{\partial N_t} \leq 0 \right\}$.

V.   Analysis: Long Delivery Lead time Supplier

When the manager must make disassembly and purchase decisions before knowing the actual yield from the components in the returned machines, we have a very complex situation. This may occur because all or part of the disassembly-diagnose-repair process takes too long and leaves little time to place an order with the supplier and wait for the respective delivery. When it happens, the planner has to order the new parts based on estimated need. The second row in Table 2 shows three such cases, depending on when process yield is known, relative to other decision moments: before disassembly, after disassembly or after repair. In the last two cases, some shortage may occur and that must be incorporated in the cost function.

Early Yield Information

If the production planner acquires the yield information before selecting the number of cores to disassemble, the scenario with "early yield information" is characterized. This situation prevents shortage of all parts, as long as there are enough cores to disassemble. However, yield is not available early enough to help in selecting the number of new parts to order from the long lead time supplier. This is the sequence of events:

Place purchase order for x_i parts based on estimates of the process recovery yield.

Learn yield y_i of each part for the disassembly kit.

Disassemble N used machines, discarding the components that will not be repaired and reused.

Repair q_i of each reparable part with the purpose to obtain D kits for the reassembly process.

Receive x_i parts ordered from new-parts supplier.

Deliver exactly D units of each part to the reassembly line and incur some holding cost.

The inventory of repaired or new parts and the inventory of reparable parts at the end of the period depend on the number of cores disassembled in the beginning of the period, shown in Figure 3. The disassembly lot size (N_t) depends on the size of the purchase order (X_{it}) made earlier in the period and on the recovery yield (Y_{it}) just learned. These variables are key to select the number of parts of each type to repair. First, it is necessary to define *critical lot*: the smallest disassembly lot size that provides a sufficient number of reparable parts of a given type. The largest critical lot guarantees meeting total demand, and it satisfies the expression

$$N_t = \max_i \left\{ \left\lceil \frac{D - \omega_{i,t-1} - m_{i,t-1}}{y_{it}} \right\rceil \right\}$$

The downside of this lot size is an ever-increasing inventory of those reparable parts with large expected yield. To counter this problem, the planner should buy more new parts with low yield, and buy fewer (or no) parts with high recovery yield. Let $\bar{y}_i$ designate the expected value of the yield for part *i*. Then, for a given lot size (N_t), the purchase size to prevent shortage would be

$$x_{it} = \left(D - \omega_{i,t-1} - \left(m_{i,t-1} + N_t \bar{y}_i \right) \right)^+ \tag{19.}$$
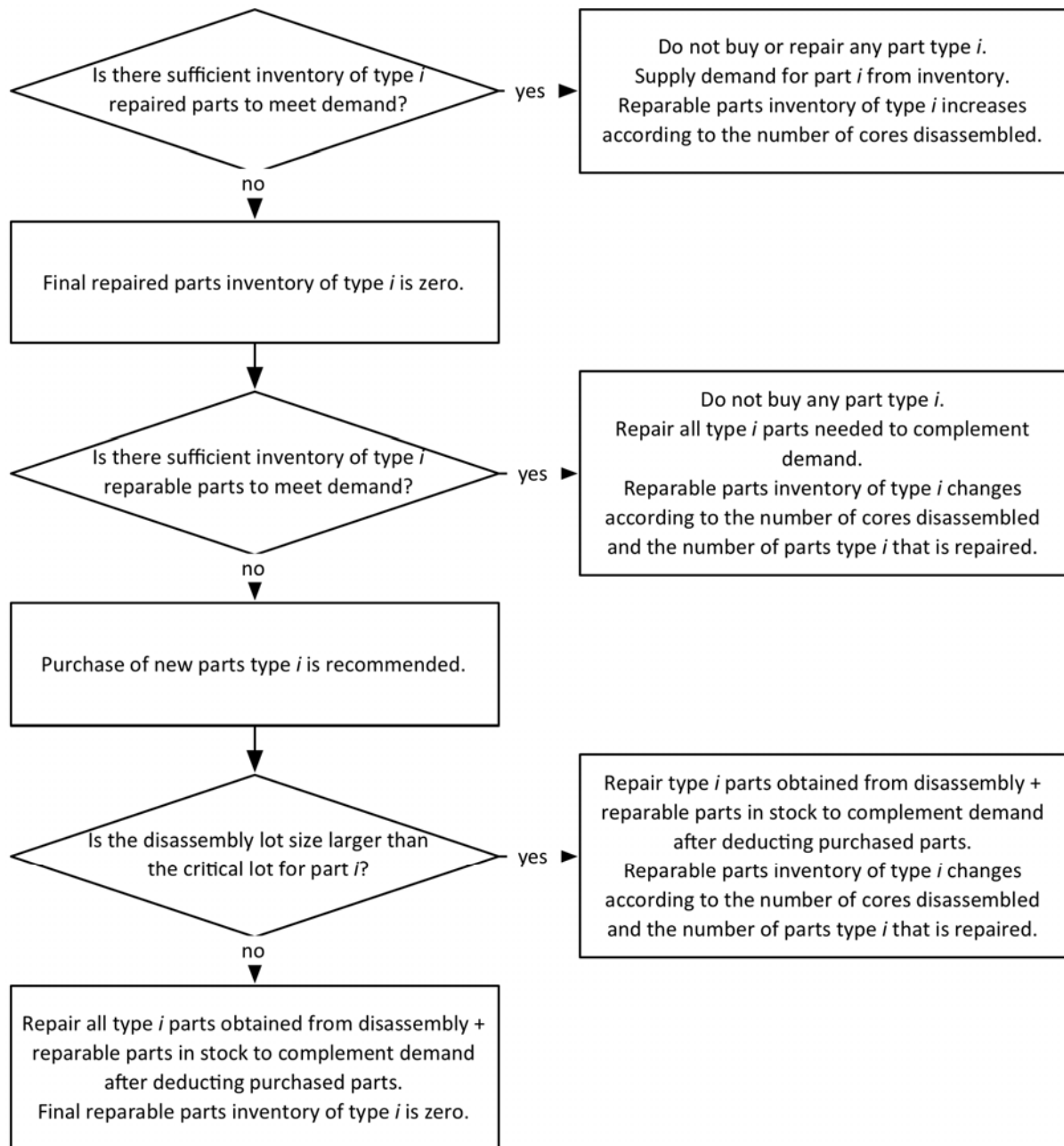
Figure 3. Repair Policy in Scenario A-2

If the actual yield turns out to be the same as the expected yield, then equation (19) would provide the optimal purchase order for all part types. However, the planner selects x_{it} before y_{it} is known and N_t is selected. If the actual yield for part i is higher than expected, there is excess inventory of that part. If actual yield is lower than expected, additional cores need to be disassembled to complement the

kit for the production process. Using the order size in (19), the approximate expected cost is this expression:

$$C(N_t) \approx kN_t + \sum_i \left[p_i \left(D - \omega_{i,t-1} - \left(m_{i,t-1} + \bar{y}_i N_t \right) \right) + r_i \left(m_{i,t-1} + \bar{y}_i N_t \right) \right]$$

$$+ \sum_i \left[h_i \int_{\bar{y}_i}^1 (y - \bar{y}_i) N_t dF_i(y) + \frac{2k}{z} \int_0^{\bar{y}_i} \frac{\bar{y}_i - y}{y} N_t dF_i(y) \right]$$

(20.)

The first integral estimates the holding cost for the residual reparable parts in this cycle. They result from higher-than-average yield. The second integral estimates the additional units that need to be disassembled, caused by parts that achieve lower-than-average yield. Remember that only those parts satisfying $D > \omega_{i,t-1} + m_{i,t-1}$ should be included in equation (20). In any one cycle, approximately half of the parts is responsible for the holding cost, and the other half is responsible for the additional disassembly cost, which explains the holding cost approximation. The gradient of the cost function is

$$\frac{\partial C(N_t)}{\partial N_t} \approx k + \sum_i \left[\bar{y}_i (r_i - p_i) + h_i \int_{\bar{y}_i}^1 (y - \bar{y}_i) dF_i(y) + \frac{2k}{z} \int_0^{\bar{y}_i} \frac{\bar{y}_i - y}{y} dF_i(y) \right]$$

(21.)

If the gradient is positive, then no core should be disassembled ($N_t^* = 0$) and

$$x_{it}^* = D - \omega_{i,t-1} - m_{i,t-1}$$

(22.)

$$q_{it}^* = m_{i,t-1}$$

(23.)

Moreover, the residual inventory of repairable and repaired components is zero $\left(m_{it} = \omega_{it} = 0 \right)$. The updated inventory levels of parts that were fully supplied at the beginning of the cycle are $\omega_{it} = \left(\omega_{i,t-1} - D \right)^+$ and $m_{it} = m_{i,t-1} - \left(D - \omega_{i,t-1} \right)^+$.

If the gradient is not positive, no new parts should be purchased ($X_t^* = 0$), and the optimal policy is

$$N_t{}^* = \max_i \left\{ \left\lceil \frac{D - \omega_{i,t-1} - m_{i,t-1}}{y_{it}} \right\rceil \right\} \tag{24.}$$

$$q_{it}{}^* = \left(D - \omega_{i,t-1} \right)^+ \tag{25.}$$

In this case, $\omega_{it} = \left(\omega_{i,t-1} - D \right)^+$ and $m_{it} = \left(m_{i,t-1} + y_{it} N_t{}^* - \left(D - \omega_{i,t-1} \right) \right)^+$. Given the nature of the "bang-bang" solution, it is going to be optimal for most parameter sets, which can be confirmed through simulation.

Yield Information upon Disassembly

If the production planner acquires the yield information before selecting the number of cores to recover, the scenario with "yield information upon disassembly" is characterized. Since the planner has to order new parts and choose the disassembly lot size before knowing yield, it is possible for shortage to occur in some cycles. This is the sequence of events:

Place purchase order for x_i parts based on estimates of the process recovery yield.

Disassemble N used machines, also based on estimates of the process recovery yield, discarding the components that cannot be repaired and reused.

Learn yield y_i of each part during disassembly.

Observe a shortage of Θ kits due to overestimating the recovery yield of some parts.

Repair q_i of each reparable part with the purpose to obtain $D - \Theta$ kits for the reassembly process.

Receive x_i parts ordered from new-parts supplier.

Deliver exactly $D - \Theta$ units of each part to the reassembly line.

The finished product shortage (Θ_t) is determined by the part with the largest shortage in the cycle. The repair lot size (Q_t) depends on the recovery yield (Y_t) and on the finished product shortage, learned after disassembly. Figure 4 shows the sequence of events following disassembly. If the actual yield for part i is higher than

expected, then there is excess inventory of that part. If actual yield is lower than expected, then there could be a shortage. Therefore, the total cost is the sum of several distinct cost components, as follows:

$$C(N, X) = kN_t + \sum_i \left[p_i x_{it} + r_i q_{it} + g_i \omega_{it} + h_i m_{it} \right] + s\Theta_t \qquad (26.)$$

The planner should trade-off the costs of incurring a shortage with the cost of ordering more new parts than the expected demand. The expected value of the shortage cost depends on the convolution of the yield distribution of all parts. Rather than expanding the exact expression (which is not tractable), the following approximation of the expected total cost helps finding an efficient policy:

$$C(N_t, X_t) \approx kN_t + \sum_i \left[p_i x_i + h_i \int_{\frac{D - \omega_{i,t4} - m_{i,t4} - x_{it}}{N_t}}^{1} \left(\omega_{i,t-1} + m_{i,t-1} + x_{it} + yN_t - D \right) dF_i(y) \right]$$

$$+ \sum_i \left[r_i \left(\int_0^{\frac{D - \omega_{i,t4} - m_{i,t4} - x_{it}}{N_t}} \left(m_{i,t-1} + yN_t \right) dF_i(y) + \int_{\frac{D - \omega_{i,t4} - m_{i,t4} - x_{it}}{N_t}}^{1} \left(D - \omega_{i,t-1} - x_{it} \right) dF_i(y) \right) \right] \qquad (27.)$$

$$+ \sum_i \left[\frac{2s}{z} \int_0^{\frac{D - \omega_{i,t4} - m_{i,t4} - x_{it}}{N_t}} \left(D - \omega_{i,t-1} - m_{i,t-1} - x_{it} - yN_t \right) dF_i(y) \right]$$

The first integral estimates the holding cost for reparable parts that are not used in this cycle. The second and third integrals estimate the repair cost for both ranges of process yield: below and above the critical value. The last integral estimates the shortage cost. This simplification is similar to the one in equation (20): in any set of z distinct parts with uncorrelated yield distribution, half of them are likely to have lower-than-average yield. If the limits of integration are close to the distribution median, then each part that is short is responsible for approximately $2/z$ of the total shortage. The gradients of the approximate cost function are given by the following expressions:
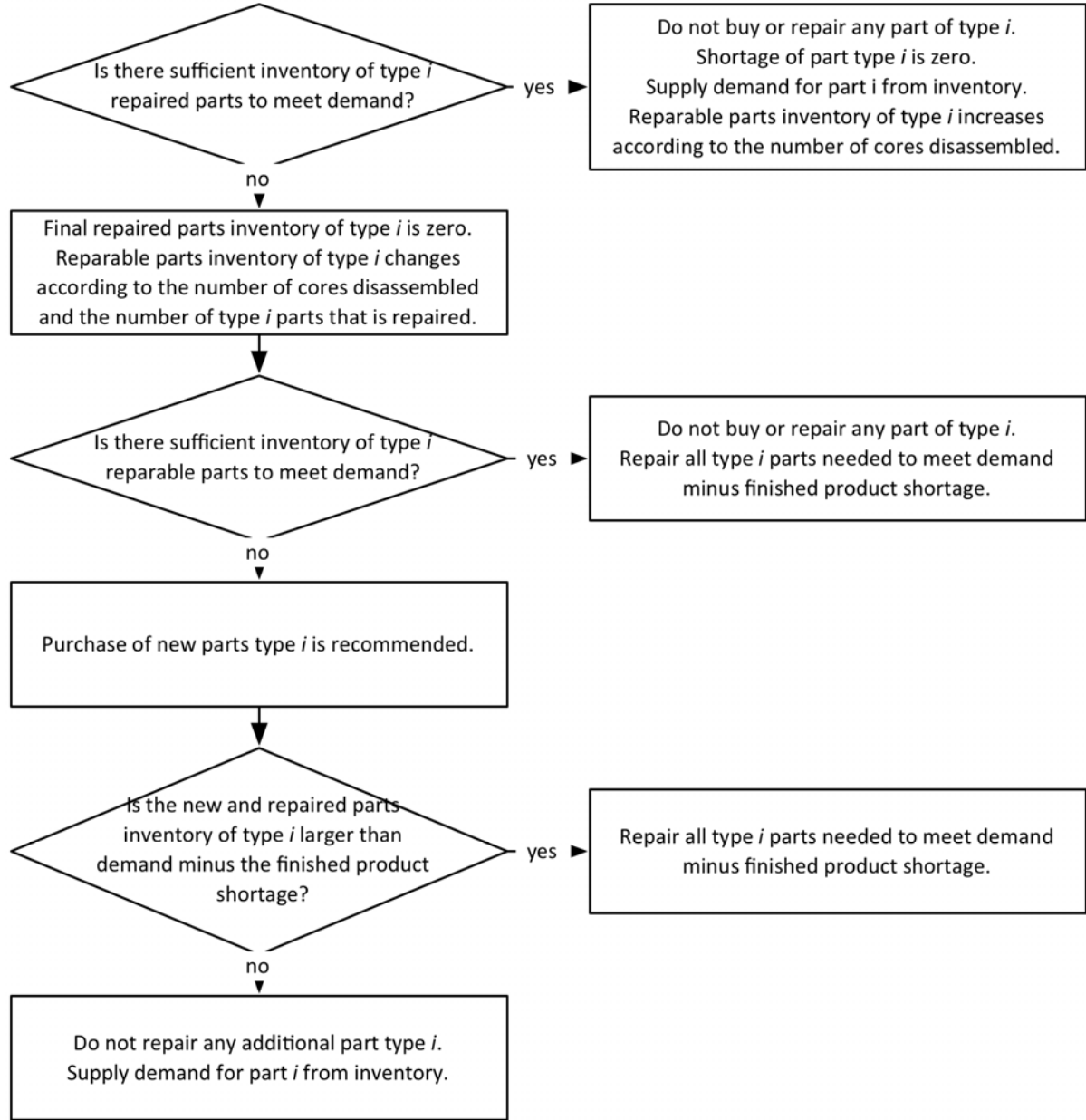
Figure 4. Repair Policy in Scenario B-2

$$\frac{\partial C(N_t, X_t)}{\partial N_t} \approx k + \sum_i \left[h_i \int_{\frac{D-\omega_{i,t4}-m_{i,t4}-x_{it}}{N_t}}^{1} y dF_i(y) - \left(\frac{2s}{z} - r_i\right) \int_{0}^{\frac{D-\omega_{i,t4}-m_{i,t4}-x_{it}}{N_t}} y dF_i(y) \right] \quad (28.)$$

$$\frac{\partial C(N_t, X_t)}{\partial x_{it}} \approx p_i - (r_i - h_i)\int_{\frac{D-\omega_{i,t4}-m_{i,t4}-x_{it}}{N_t}}^{1} dF_i(y) - \frac{2s}{z}\int_0^{\frac{D-\omega_{i,t4}-m_{i,t4}-x_{it}}{N_t}} dF_i(y)$$

$$= p_i + h_i - r_i - \left(h_i - r_i + \frac{2s}{z}\right)F_i\left[\frac{D-\omega_{i,t-1}-m_{i,t-1}-x_{it}}{N_t}\right]$$

(29.)

The approximate cost function in equation (27) is convex, so the (N_t, X_t) array that zeroes equations (28) and (29)—if it exists—minimizes the cost function. Such gradient may not be unique. Define $\xi_{it} = (D - \omega_{i,t-1} - m_{i,t-1} - x_{it})/N_t$ as the ratios that zero the right-hand side in equation (29). For a given part i, the ratio associates the disassembly lot size with the purchase order x_{it} that minimizes the total cost of obtaining that part. The following heuristic finds the (N_t, X_t) array that minimizes the approximate cost function in a few steps:

Identify the ratio ξ_{it} for each part.

Identify the disassembly lot size n_{it} corresponding to ξ_{it} when $x_{it} = 0$.

Let $\varphi = \min_i \{ n_{it} \}$.

Identify the value $x_{it} \geq 0$ corresponding to ξ_{it} when $N_t = \varphi$. For any part of type i, if only a negative value of x_{it} can zero the equation, let $x_{it} = 0$.

Let $\Xi = \{ x_{it} \}$, a matrix of all x_{it} found in step 4.

Identify the disassembly lot size N_t that zeroes equation (28) when $X_t = \Xi$.

Let $\varphi = | N_t$

Perform a convergence test. If φ did not converge, return to step 4, using the current value of φ. Otherwise, let $N_t^* = \varphi$, $X_t^* = \Xi$ and stop.

Considering the shape of the cost function, convergence should happen in very few cycles. Repair quantity is $q_{it}^* = \min\left\{ y_{it}N_t^* + m_{i,t-1}, (D - x_{it}^* - \omega_{i,t-1} - \Theta_t)^+ \right\}$. If any part experiences shortage, then it will be $\theta_{it} = ((D - \omega_{i,t-1} - x_{it}^*) - (m_{i,t-1} + y_{it}N_t^*))^+$, and

the number finished products not delivered will be $\Theta_t = \max_i(\theta_{it})$. Finally, the residual

inventories are $m_{it} = \left(m_{i,t-1} + y_{it} N_t{}^* - (D - \omega_{i,t-1} - x_{it}{}^* - \Theta_t)^+ \right)^+$ and $\omega_{it} = (\omega_{i,t-1} + x_{it}{}^* - D + \Theta_t)^+$.

Simulation shows that the resulting $(N_t{}^*, X_t{}^*)$ array provides a good solution to the

actual cost function.

Yield Information upon Component Repair

If a quick diagnosis is not able to indicate whether the component can be recovered, the scenario with "yield information upon component repair" is characterized. Managing this scenario requires a substantial inspection process with each part before identifying its suitability for recovery, which would require making all production decisions based on the estimated yield. This is the sequence of events:

Place purchase order for x_i parts based on estimates of the process recovery yield.

Disassemble N used machines based on estimates of the process recovery yield.

Repair q_i of each reparable part to obtain D units for the reassembly process, based on estimates of the process recovery yield.

Learn yield y_i of each part during repair.

Observe a shortage of Θ kits due to overestimating the recovery yield of some parts.

Receive x_i parts ordered from new-parts supplier.
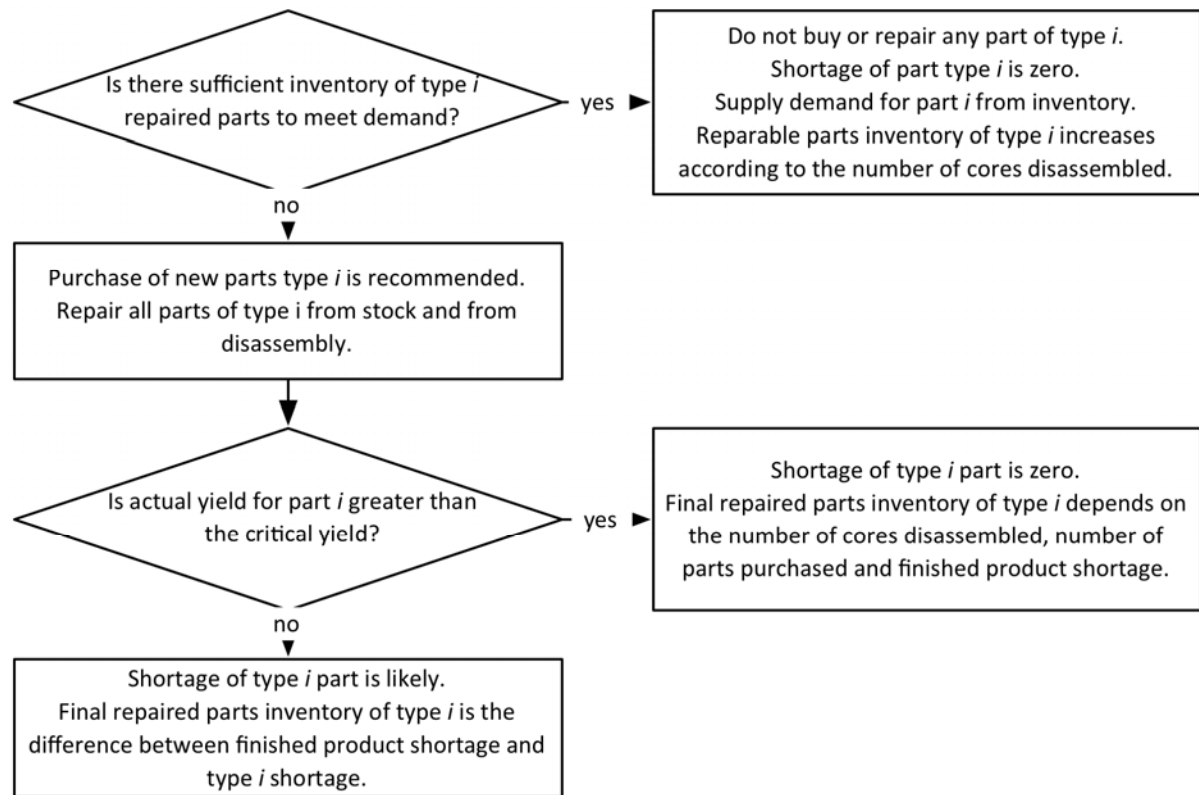
Figure 5.   Final Inventory and Shortage Level in Scenario C-2

The total cost is derived from Equation (26). However, once again, the expected value of the shortage level leads to an intractable expression, so we adopt an approximation similar to the one in the previous section, which presumes that approximately half of all parts are responsible for the shortage.

Since the planner does not know in advance the yield of each part until it is repaired, it is preferable to repair all disassembled parts issued from disassembly. As a result, the repair cost would be incurred for each disassembled part that goes into the repair process, regardless if the repair is successful or not. Only parts that have a stock of repaired parts sufficient to meet this period's demand would not repair the disassembled parts, as shown in Figure 5. However, the stock of reparable parts is the consequence of past disassembly decisions, so the associated expenses are sunk. Hence, the holding cost includes just the repaired parts that

obtained higher–than-average yield. The approximation of the expected total cost follows:

$$C(N_t, X_t) \approx kN_t + \sum_i \left| p_i x_{it} + r_i(m_{i,t-1} + N_t) + g_i \int_{\frac{D - \omega_{i,t4} - x_{it}}{m_{i,t4} + N_t}}^1 \left(\omega_{i,t-1} + x_{it} + y(m_{i,t-1} + N_t) - D\right) dF_i(y) \right|$$
$$+ \sum_i \left[\frac{2s}{z} \int_0^{\frac{D - \omega_{i,t4} - x_{it}}{m_{i,t4} + N_t}} \left(D - \omega_{i,t-1} - x_{it} - y(m_{i,t-1} + N_t)\right) dF_i(y) \right] \tag{30.}$$

The first integral estimates the holding cost for repaired parts that are not used in this cycle. The second integral approximates the shortage cost by sharing the burden among half of the items in the product. We apply Leibnitz's differentiation rule to find the gradient of this cost function:

$$\frac{\partial C_t(N_t, X_t)}{\partial N_t} \approx k + \sum_i \left| r_i + g_i \int_{\frac{D - \omega_{i,t4} - x_{it}}{m_{i,t4} + N_t}}^1 y \, dF_i(y) - \frac{2s}{z} \int_0^{\frac{D - \omega_{i,t4} - x_{it}}{m_{i,t4} + N_t}} y \, dF_i(y) \right| \tag{31.}$$

$$\frac{\partial C_t(N_t, X_t)}{\partial x_{it}} = p_i + g_i - \left(g_i + \frac{2s}{z}\right) F_i\left(\frac{D - \omega_{i,t-1} - x_{it}}{m_{i,t-1} + N_t}\right) \tag{32.}$$

The cost function in equation (30) is convex. So, it is necessary to find the (N_t, X_t) array that zeroes equations (31) and (32)—if it exists—to minimize the approximate cost function. Define $\xi_{it} = (D - \omega_{i,t-1} - x_{it})/(m_{i,t-1} + N_t)$ as the ratios that zero the right-hand side in equation (32). These ratios identify the array of purchase orders that minimize total cost for a given disassembly lot size. Likewise, we use equation (31) to find the disassembly lot size N_t that minimizes total cost for a given array of purchase orders. In the previous scenario (case B-2) a heuristic was used to find the (N_t^*, X_t^*) array that minimizes the approximate cost function in just a few steps. The same heuristic can be used here with success.

Repair quantity is $q_{it}^* = N_t + m_{i,t-1}$. If any part experiences shortage, then it will

be $\theta_{it} = \left(\left(D - \omega_{i,t-1} - x_{it}^* \right) - y_{it} \left(m_{i,t-1} + N_t^* \right) \right)^+$, and the number of finished products not

delivered will be $\Theta_t = \max_i \left(\theta_{it} \right)$. Finally, the residual inventories are $m_{it} = 0$ and

$\omega_{it} = \left(\omega_{i,t-1} + y_{it} \left(N_t - m_{i,t-1} \right) - D + \Theta_t \right)^+$. Simulation shows that the resulting $\left(N_t^*, X_t^* \right)$ array

provides a good solution to the *actual* cost function. Simulation shows that the
resulting cost is very close to optimality.

VI. Conclusion

The production plan of remanufacturing operations has eluded researchers for almost two decades. There are several variables that can alter process behavior and the decision-making process, making it difficult to define an optimal policy that is suitable to all remanufacturing operations. Past research identified four types of collection processes, to which this study adds two more types: "Planned Upgrades" and "Collection for Recovery," leading to six motivations for used product collection.

This study discusses the impact of information availability in each stage of the remanufacturing process, focusing on products that are complex and generally built using a modular design. Our assumptions are consistent with the product returns described as "End-of-use Returns," "Commercial Returns," "Planned Upgrades," and "Collection for Recovery." Military assets that are remanufactured frequently fall into the latter categories.

This research demonstrates that the production plan is relatively simple whenever the decision-maker can rely on suppliers that deliver the necessary parts for remanufacturing in relatively short lead times. It provides optimal policies to three scenarios in which the responsible supplier is capable of making on-time deliveries, even when the purchase order is placed after the parts recovery yield is identified, the cases in row A of Table 2. It also provides near-optimal policies for the cases in which the supplier requires longer lead times, i.e., the decision-maker must place the purchase order before identifying precisely the recovery yield of the significant parts in the product. These near-optimal policies have shown excellent performance in simulated tests. Further research should evaluate how these policies behave in a dynamic multi-cycle remanufacturing operation.

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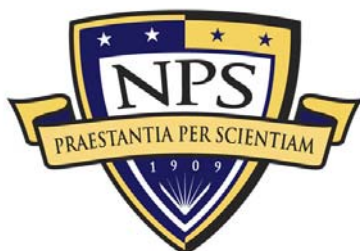
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